Physics 457, Winter 2005
Midterm Exam I

This exam is due at 5 PM Friday, March 11, 2005, in Chupp's office. You may use reference materials, books, notes and the internet. Do not work with or consult others regarding this exam.

1. The π^- meson at rest in the lab decays into a negative muon and an antineutrino.

$$m_{\pi^\pm}\ c^2 = 139.6 \text{ MeV and } m_\mu\ c^2 = 105.7 \text{ MeV.}$$

Consider the neutrino to be masssless and therefore relativistic.

a.) Write down the π^- wave function, properly normalized, correctly including spin, isospin, and color components.

b.) The antineutrino spin is **parallel to** its momentum, that is its helicity $h_\nu = \hat{s}_\nu \cdot \hat{p}_\nu = +1$. What is the helicity of the muon $h_\mu = \hat{s}_\mu \cdot \hat{p}_\mu$?

c.) What is the kinetic energy of the muon?

2. Consider the proton, a $J^\pi = 1/2^+$ mixed-symmtery baryon octet member.

a.) What are the other octet members?

b.) Write the properly normalized wave-function of a spin up proton including the isospin-spin, spatial, and color components and show that it is indeed antisymmetric under exchange of any two quarks.

Note that the isospin-spin component has terms $|u^\uparrow\ u^\uparrow\ d^\downarrow >$, $-|u^\uparrow\ u^\downarrow\ d^\uparrow >$ and all appropriate permutations (12 terms in all).

c.) Recall the magnetic moment operator for a spin-1/2 quark is:

$$\vec{\mu}_f = \frac{1}{m_f} Q\vec{s}$$

where m_f is the mass of a quark of "flavor" f and Q is the charge operator, e.g. $Q|d> = -1/3e|d>$.

Use this operator to find the expectation value of the z–component of the magnetic moment of a spin-up proton, $<\mu_p^z> = <p, m_s = +1/2|\sum_{i=1}^{3} \mu_i^z\ |p, m_s = +1/2>$.

Express this in terms of the nuclear magneton and compare your result to the measured proton magnetic moment, $\mu_p = 2.78\mu_N$.

3. A 1 GeV electron beam of with 10^{12} electrons per second is incident on a liquid hydrogen target of density $\rho = 0.07 \times 10^3$ kg/m^3 and thickness 0.5 m. A detector of 1000 cm^2 area is located 10 meters from the target at an angle of 5°.

a.) What are $|\vec{q}|$ and ν in the limit of negligible electron mass.

b.) What is the proton form factor $F(Q^2)$ for this example? (Recall $\sigma_{ep} = |F(Q^2)|^2 \sigma_{Mott}$.)

c.) Find the scattering rate for electrons in the detector.

5. Consider the charge distribution for a particle of charge e given by

$$\rho_Q(r) = \rho_0 \text{ for r} \leq \text{R}; \quad \rho_Q(r) = 0 \text{ for r} > \text{R}$$

a.) What is ρ_0?

b.) Find $F(\vec{q}^{\,2})$.

c.) What is the value of $|\vec{q}^{\,2}|$ for the first zero of $F(\vec{q}^{\,2})$ (*i.e.* the zero corresponding to the lowest value of $|\vec{q}^{\,2}|$.

d.) What is $< r^2 >^{1/2}$ in terms of R?

$$f(\vec{q}) = -\frac{m}{2\pi\hbar^2} \int V(\vec{r}) e^{i(\vec{q}\cdot\vec{r})/\hbar} d^3r \quad \vec{q} = \vec{p} - \vec{p}\,'$$

$$\sigma(\theta) = \frac{d\sigma}{d\Omega} = |f(q)|^2 = |F(q^2)|^2 \sigma_{R,\ M,\ ...} \qquad \text{where} \quad F(q^2) = \int \rho(r) e^{i(\vec{q}\cdot\vec{r})/\hbar} d^3r$$

$$\sigma_R = \frac{(ZZ_1\hbar c\alpha_e)^2}{4E_0^2 \sin^4(\theta/2)} \quad \sigma_M = 4\frac{(Z\hbar c\alpha_e)^2 E_0^2}{(pc)^4}(1 - \beta^2 \sin^2\frac{\theta}{2}) \approx \frac{(Z\hbar c\alpha_e)^2}{4E_0^2 \sin^4(\theta/2)}(\cos^2\frac{\theta}{2})$$

$$\sigma_{ep}^{elas} = \sigma_M[\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2\frac{\theta}{2}] \quad \tau = \frac{|q^2|c^2}{4m_p^2c^4}$$

$$G_E^p(q^2) = \frac{1}{(1 + (q^2/q_0^2))^2} \quad \text{and} \quad G_M^p(q^2) = \frac{\mu_p}{\mu_N} G_E(q^2)$$

$$q_0^2 = 0.71(\text{GeV/c})^2$$