Physics 457 Problem Set 4

Due in Class, February 9, 2005

Recall

$$f(\vec{q}) = -\frac{m}{2\pi\hbar^2}\int V(\vec{r})e^{i(\vec{q}\cdot\vec{r})/\hbar}d^3r \quad \vec{q} = \vec{p} - \vec{p}'$$

$$\sigma(\theta) = \frac{d\sigma}{d\Omega} = |f(q)|^2 = |F(q^2)|\sigma_{R,\ M,\ \ldots}$$

$$\text{where} \quad F(q^2) = \int \rho(r)e^{i(\vec{q}\cdot\vec{r})/\hbar}d^3r$$

$$\sigma_R = \frac{(ZZ_1\hbar c\alpha_e)^2}{4E_0^2\sin^4(\theta/2)} \quad \sigma_M = 4\frac{(Z\hbar c\alpha_e)^2E_0^2}{(pc)^4}(1-\beta^2\sin^2\frac{\theta}{2}) \approx \frac{(Z\hbar c\alpha_e)^2}{4E_0^2\sin^4(\theta/2)}(\cos^2\frac{\theta}{2})$$

$$\sigma_{ep}^{elas} = \sigma_M[\frac{G_E^2+\tau G_M^2}{1+\tau} + 2\tau G_M^2\tan^2\frac{\theta}{2}] \quad \tau = \frac{|q^2|c^2}{4m_p^2c^4}$$

1. (Continued from P.S. 3) Numerically evaluate the elastic cross section for scattering of 800 MeV electrons from protons at 30°.

Find the proton form factor assuming the dipole form

$$G_E^p(q^2) = \frac{1}{(1+(q^2/q_0^2))^2} \quad \text{and} \quad G_M^p(q^2) = \frac{\mu_p}{\mu_N}G_E(q^2)$$

$$q_0^2 = 0.71(\text{GeV/c})^2 \ q \text{ and } q_0 \text{ are a } 4-\text{vectors}$$

2. For deep inelastic scattering, show that with $|\vec{p}_q|c \approx E_q = x|\vec{p}_p|$ and $\vec{p}_q' = \vec{q}$

$$x = \frac{Q^2}{2m_p\nu}$$

3. For a Deep Inelastic Scattering experiment, the incident electron energy is $E_e = 20$ GeV and electrons are observed to scatter at 5°.
a.) Find Q^2 vs E_e'. Indicate E' for $x = 1$, $x - 0.5$, $x = 0.2$, and $x = 0.1$. Elastic scattering, of course, corresponds to $x = 1$. b.) Assume that the scattering process is dominated by scattering from a single quark and that there are equal numbers of up quarks and down quarks. Estimate the cross section $\frac{d\sigma}{d\Omega}$ assuming the form factor is independent of q^2.

4. Show by explicit calculation in spherical coordinates that the parity operator P ($\vec{r} \rightarrow -\vec{r}$) has the effect $\theta \rightarrow \pi - \theta$ and $\phi \rightarrow \pi + \phi$. Also show $P^2 = 1$

5. Recall that the spatial part of the wave function of a system that conserves orbital angular momentum (*i.e.* $[H, L] = 0$) can be separated into a radial part an angular part $Y_{l,m}(\theta, \phi)$. Show that $P\ Y_{l,m} \ = \ (-1)^l\ Y_{l,m}(\theta, \phi)$ for $l = 0$, 1, and 2. (You may also show that this is true in general, *i.e.* for all l.)