Physics 457 Problem Set 5

Due in Class, February 16, 2005

1. (Postponed from last week) Show explicitly that the $q = 0$ pseudoscalar mesons are eigenstates of C with eigenvalue $C = +1$, where

$$|\pi^0 >= \frac{1}{\sqrt{2}}(|u\bar{u} > -|d\bar{d} >) \quad |\eta >= \frac{1}{\sqrt{6}}(|u\bar{u} > +|d\bar{d} > -2|s\bar{s} >)$$

$$|\eta' >= \frac{1}{\sqrt{3}}(|u\bar{u} > +|d\bar{d} > +|s\bar{s} >)$$

(Use the antisymmetry of the quark–fermion wavefunction, and recall that the singlet spin wave function is **antisymmetric** under exchange.)

2. The meson mass formula is

$$m(q_1\bar{q}_2) = m_1 + m_2 + \frac{a}{m_1 m_2} < s|\vec{s}_1 \cdot \vec{s}_2| >$$

a.) Calculate explicitly the matrix element $< s|\vec{s}_1 \cdot \vec{s}_2| >$ for singlet or scalar ($s = 0$) and triplet or vector ($s = 1$) mesons.
b.) Use $m_u = m_d = 310$ MeV (*i.e.* constituent masses) and the ρ – π mass difference to estimate a. ($m_\rho = 770$ MeV and $m_\pi = 135$ MeV.)

3. Consider the bound states of Charmonium with $n = 1$ and $n = 2$.
a.) List all of the possible states and their quantum numbers including J^{PC} and $n^{(2s+1)}l_j$. There are 2 $n = 1$ levels (singlet and triplet) and a total of 6 $n = 2$ levels.
b.) Identify the η_c (2980 MeV), J/ψ (3097 MeV), ψ (3685 MeV), χ_{c0} (3415 MeV), χ_{c0} (3415 MeV), and χ_{c0} (3415 MeV).

4 a.) Use the $\psi(2s)$ to $J\psi(1s)$ mass splitting to determine α_s.

$$\text{Assume} \quad V(r) = -\hbar c\frac{\alpha_s}{r}$$

Hint: The positronum energy levels, without hyperfine splitting, are given by

$$E_n = \frac{m_e c^2 \alpha_e^2}{2n^2}$$

where $\hbar c\alpha_e = ke^2$.
b.) What is the effective Bohr radius for Charmonium?