Modern Subatomic Physics

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Chapter 1

Introduction

1.1 The meaning of "Subatomic Physics"

This course is about physics on the scale of the atomic nucleus, that is on the order of 10^{-15} m (or 1 fm), and smaller. At this "subatomic scale," the players are nuclei, nucleons (protons and neutrons), leptons (such as electrons and neutrinos), quarks, and the hadrons (baryons and mesons) made up of them. This long list of players results from the composite nature of hadrons, including nucleons, and nuclei. In fact a coherent picture, called the standard model, results from considering the quantum mechanics of just two classes of elementary, that is structureless, particle: leptons and quarks.

The study of subatomic physics is concerned with the following:
1. The constituents of nuclei and nucleons
2. Their strucutre, that is the size, shape and distribution of charge and mass as well as the distribution of energy levels.
3. Their interactions which include strong, electromagnetic and weak (or electroweak)
4. Applications, which means both practical applications and applications of "the nucleus as a laboratory." The example of meaurement of solar neutrino flux demonstrates how the constituents, structure and interactions of nuclei are applied to the fundamental problem of measuring the production of energy in the sun.

1.2 The meaning of "scale"

Understanding the role that the scale of a physical system or a physics problem is a crucial goal of this course. The advances in physics have resulted from isolating the scale of the problem under consideration. For example, to study the solar system we do not look at planets as collections or quarks or even atoms. This is not a useful scale, and any force other than gravity is not relevant. To study an atom, we do best considering its scale and the electromagnetic interaction ignoring, for example the idea that quarks are constituents of the nucleons. This isolation of scale is not perfect: there are generally small scale effects in a system. For an atom, the structure of the nucleus plays a role in the structure of the electronic energy levels. For hydrogen this is typically a few parts in 10^5, the ratio of the atom's size to that of the nucleus.

The relationship of the distance scale to the energy scale of a system can be understood by considering how it is we learn about the structure of a system. In subatomic physics experiments, structure is generally probed by scattering a probe of well understood particles, such as leptons (*i.e. electrons or neutrinos* from the system of interest. In such cases, the momentum transfered between the probe and system (labeled **q**) [1] has a deBroglie wavelength (divided by 2π)

$$\lambda\!\!\!^{-} = \frac{\hbar}{q} = \frac{\hbar c}{qc}.$$

The quantity $\hbar c = 197.3$ MeV-fm will be encountered repeatedly. It sets the energy scale of physics at the femtometer distance scale at 200 MeV = 0.2 GeV. Thus to probe the structure of the nucleus, probes of energy several hundred MeV to GeV are necessary.

The following compares the scale of the atom to that of the nucleus and nucleons.

The Energy and Distance Scales of Atoms and Nuclei

System	Size	Energy Scale	Typical Binding Energy (per particle)
Atom	few Å(10^{-10}) m	1 keV	10 eV
Nucleus	few fm (10^{-15}) m	100 MeV	10 MeV
Nucleon	1 fm	> 0.2 GeV	[2]

1.3 Subatomic Physics Applied to the Observation of Solar Neutrinos

(See section 19.2 of Frauenfelder and Henley)

Stars including the sun are believed "burn" hydrogen in their cores through a series of nuclear reactions that can be summarized as

$$4\ p\ \rightarrow\ {}^4\mathrm{He}\ +\ 2e^+ +\ 2\nu \qquad (1)$$

Here ^{4}He refers to the nucleus of helium, often referred to as the α particle. For every four protons converted into one helium nucleus, 26.7 MeV of energy is produced. The idea that this is the dominant stellar energy process was suggested originally by Eddington in the 1920's.

In more detail, there are several multi-step chains that can be summarized by equation 1. The most prevelant is:

$$p\ +\ p\ \rightarrow\ d\ +\ e^+\ \nu_e\ \mathrm{or} p\ +\ p +\ e^-\ \rightarrow\ d\ + \nu_e$$

$$2 \times (d\ +\ p\ \rightarrow\ {}^3\mathrm{He}\ + \gamma$$

$$^3\mathrm{He}\ +\ {}^3\mathrm{He}\ \rightarrow\ {}^4\mathrm{He}\ +\ 2p$$

[1]I will indicate vectors in **bold faced print**

Another path is ^{3}He–^{4}He fusion:

$$^3\text{He} \ + \ ^4\text{He} \ \rightarrow \ ^7\text{Be} \ + \ \gamma$$

$$^7\text{Be} + e^- \ \rightarrow \ ^7\text{Li} \ + \ \nu$$

$$^7\text{Li} + p \ \rightarrow \ 2\ ^4\text{He}$$

$$\text{or} \quad ^7\text{Be} + p \ \rightarrow \ ^8\text{Li} \ + \ \gamma$$

$$^8\text{B} \ \rightarrow \ ^8\text{Be}^* \ + \ e^+ \ \nu_e$$

$$^8\text{Be}^* + p \ \rightarrow \ 2\ ^4\text{He}$$

There are several kinds of nuclear reaction involved. Those that involve neutrinos are *weak* interactions such as β decay of ^{8}B. Weak means slow, or short range, and two crucial consequences arise from the weak interaction steps in the solar energy cycle: the very first step $p \ + \ p \ \rightarrow \ d \ + \ \nu_e$ or the *pep* reaction controls the rate of conversion of hydrogen (*i.e.* protons) into helium. The consequences of a faster burning sun include more heat and a shorter lifetime. Also important is that the neutrinos produced at various steps can emerge from the core of the sun with very little probability of interacting [3] and therefore provide a peak at this series of nuclear reaction processes. The problem is detecting the solar neutrinos.

The spectrum of neutrino energies produced by this series of nuclear reactions is illustrated in Figure 1. Note that there are discrete lines, for example from ^{7}Be electron capture, and $pp \rightarrow de^+\nu$ reaction involving

Several schemes are now in use to detect the neutrinos believed to be produced in the solar core. There are two distinct techniques, one involving **inverse beta decay**, specifically

$$\nu_e \ + \ ^{37}\text{Cl} \ \rightarrow \ e^- \ ^{37}Ar \quad \text{for } \text{E}_\nu \ > \ 0.814 \text{ MeV}$$

and

$$\nu_e \ + \ ^{71}\text{Ga} \ \rightarrow \ e^- \ ^{71}Ge \quad \text{for } \text{E}_\nu \ > \ 0.233 \text{ MeV}$$

and the other involving the detection of Cerenkov radiation in water or heavy water following the recoil of an electron in the reactions

$$\nu_e \ + e \ \rightarrow \ e^- \ + \ \nu$$

or

$$\nu_e \ + d \ \rightarrow \ e^- \ + \ 2p$$

and

$$\nu_x \ + e \ \rightarrow \ \nu_x \ + \ p \ + \ n$$

Here I have been careful to distinguish electron neutrinos ν_e from other neutrino flavors related to muon and tau leptons. The integrity or conservation of lepton flavors will be discussed in lecture 3. All of these reactions except the last are insensitive to antineutrinos, and each reaction has a different threshold neutrino energy, also indicated in Figure 1.

[3] though a compelling, unconfirmed theory, suggests that this may not be true, rather that the density of electons in the sun may cause the electron neutrinos to convert (or oscillate) into another neutrino species.

SNUs5.in3.5in

Figure 1.1: Calculated solar neutrino fluxes.

These weak interactions are so weak that the probability of a neutrino interacting is best expressed as a mean free path: typically 10^{19} m for water. A light year is 9.6×10^{15} m! Only the fact that approxiamtely 10^{11} solar neutrinos per cm^2 rain down on us allows the measurements of Ray Davis and others. The Davis experiment uses ^{37}Cl, which is 25% of natural chlorine. The chlorine is part of the molecule C_2Cl_4, and 390000 liters are stored in a tank in a gold mine in South Dakota (the Homestake Mine in Lead. Davis devised a scheme to recover a very few atoms of ^{3}7Ar from the tank by bubbling argon gas through the tank as a carrier. ^{37}Ar is radioactive, decaying by the capture of an atomic electron, usually from an inner, K shell. The neutral daughter atom, ^{37}Cl is thus left in an unstable atomic state that decays by emission of a K x-ray or emissionof an outer shell electron called an Auger electron. It is the efficient detection of the Auger electron that is used to count the number of 35 day half-life ^{37}Ar atoms produced.

The solar neutrino detection rate is expressed in Solar Nuetrino Units or SNUs. One SNU is 1 event per second per 10^{3}6 target atoms. Theoretical predictions of the rate in SNUs of a specific detection scheme are based on the following:
1. A solar model describing the hydrostatic equilibrium between gravitational collapse, thermal expansion, and energy transport between layers of the sun.
2. Measurements and calculations of the branching ratios for the different paths of hydrogen burning.
3. The complete neutrino detection efficiency of the detector.
4. Finally it is assumed that once and electron neutrino is produced in the sun's core, it does not decay or change into any other, non detectable neutrino species. The possibility of neutrinos changing to antineutrinos or changing flavor (for example $\nu_e - -\nu_\mu$) are called neutrino oscillations. The existence of neutrino oscillations would fundamentally current picture of elementary particle interactions.

The theoretical prediction for the ^{37}Cl experiment is 8 SNU. The experimental observation is 2.5$\pm$ 0.3 SNU. The results of the ^{71}Ga experiments (there are two)($\approx$ 75 SNU) are also inconsistent with theory (about 130 SNU). One or more of the bases for the theoretical prediction cannot be correct. Most intriguing is the possibility that neutrinos traversing the sun "precess" or oscillate from electron neutrino to another flavor. These so called neutrino oscillations are not yet on solid experimental ground.

Subatomic physics enters the solar neutrino problem in several ways, from the original proposal of hydrogen fusion to the calculation of the neutrino spectrum based on branching ratios for nuclear reactions to the operation of the solar neutrino detector. This is just a single example of the nucleus as a laboratory. [4]

[4]See the web page at **http://snodaq.phy.queensu.ca/SNO/neutrino.html** and follow the links.

Chapter 2

Observables and Symmetries

Physics at the subatomic scale is the domain of quantum mechanics, and the observables are expectation values of quantum mechanical operators. In table I present 2.1 a list of the observables of subatomic physics. Many of these such as lepton number and parity are yet to be defined. Mass/energy, momentum, and angular momentum are familiar quantities of quantum mechanical motion. We are accustomed to conservation laws in physics, *e.g.* conservation of momentum, and observables such as lepton number are also apparently conserved quantities, or constants of the motion. We begin this lecture by drawing the connection between operators and the conservation of operators' expectation values. The operators of quantum mechanics also generate transformations: the momentum operator, for example, generates translations, and the angular momentum operator generates rotations. The invariance of the physics of a system (the set of consreved observables) under a transformation is called a symmetry of the system. Symmetry in this quantum mechanical is crucial to subatomic physics.

2.1 Observables

We begin with a review of observables in quantum mechanics. This review is a statement of the axioms of quantum mechanics, with which you are familiar. A quantum mechanical observable is the expectation value of an operator, e.g. $\mathcal{A}$.

$$< \mathcal{A} >= \int_{all\ \ space} \psi^*\ \mathcal{A}\ \psi\ d^3x =< \psi|\mathcal{A}|\psi >$$

The second expression is the Dirac Braket notation and is equivalent to the integral form.
Axiom 1 ψ^* and ψ are complex wave functions that are solutions to a wave equation such as

$$i\hbar\ \frac{d\psi}{dt} = H\ \psi \quad \text{the time dependent Schrödinger equation}$$

$|\psi|^2 {=} \psi^*\psi\ \geq\ 0$ is a probability density with units $1/(volume)$.
The time dependent Schrödinger equation is a diffusion equation of the kind that describes the dynamics of a fluid, that is the probability density in quantum mechanics has the character of a fluid governed by a continuity equation relating the current density **J** to the rate

of change of $|\psi|^2$, that is the rate of change of probability within a volume element d^3x is equal to the probability flowing into that volume or

$$\nabla \cdot \mathbf{J} + \frac{d|\psi|^2}{dt} = 0$$

The diffusion equation

$$\mathbf{J} = D\nabla|\psi|^2 \quad \text{or} \quad \frac{d|\psi|^2}{dt} = D\nabla^2|\psi|^2$$

is the sum of Schrödinger equations for ψ and for ψ^*.

Axiom 2 Physical observalbes are represented by linear, hermitean operators such as $p_x = i\hbar\frac{d}{dx}$, Q (the charge operator), $\mathbf{s} = \frac{\hbar}{2}\sigma$, and $\mathbf{L} = \mathbf{r} \times \mathbf{p}$.

Axiom 3 The eigenvalue equation for an operator $\mathcal{A}$ is

$$\mathcal{A}\Phi_n = a_n\Phi_n$$

where Φ_n is the nth eigenstate of $\mathcal{A}$.

Axiom 4 The set of eigenstates Φ_n form a complete, orthonormal set, that is

$$\psi = \sum_n \alpha_n\Phi_n \quad \text{and} \quad <\Phi_n|\Phi_m>= \delta_{n,m}$$

Axiom 5 Following from above

$$<\mathcal{A}>_\psi =<\psi|\mathcal{A}|\psi>= \sum_n |\alpha_n|^2 a_n$$

Axiom 6 The time dependence of an operator $<\mathcal{A}>$ with no explicit time dependence is given by

$$i\hbar\frac{d}{dt} <\mathcal{A}>= i\hbar\frac{d}{dt}\int \psi^*\mathcal{A}\psi\ d^3x = i\hbar\int [\psi^*\mathcal{A}H\psi - \psi^*H\mathcal{A}\psi]\ d^3x$$

which follows from the time dependent Schrödinger equations for ψ and ψ^* and the hermiticity of H, i.e.

$$\int (H\psi)^*\psi\ d^3x = \int \psi^*H\psi\ d^3x.$$

From this it follows that

$$i\hbar\frac{d}{dt} <\mathcal{A}>=<\mathcal{A}H - H\mathcal{A}>=<[\mathcal{A},H]> .$$

A special situation arises when $[\mathcal{A},H] = 0$: The expectation value $<\mathcal{A}>$ is a constant, conserved quantitity.

Observables of Subatomic Physics - not all strictly conserved

Quantity	Operator	Symmetry transformation
Mass/Energy	H	time translation
Charge	Q	gauge transformation
Spin Angular Momentum	$\mathbf{s} = \frac{\hbar}{2}\sigma$	rotations in spin space
Orbital Angular Momentum	$\mathbf{L} = \mathbf{r} \times \mathbf{p}$	rotations
Total Angular Momentum	$\mathbf{J} = \mathbf{L} + \mathbf{S}$	generalized rotations
Isospin	$\mathbf{I} = \frac{1}{2}\sigma$	rotations in spin space
Lepton number	L	
Lepton flavor	L_e, L_μ, L_τ	
Baryon number	B	
Quark flavor	L_{ud}, L_{cs}, L_{tb}	
Parity	P	parity reversal
Charge Conjugation	C	particle $\longleftrightarrow$ anti-particle
Parity	P	parity reversal

Other properties, such as decayu rate or lifetime, are not quantum mechanical observables in the same sense as those listed in table 2.1.
Notes The components of **J** (and **L** and **s**) satisfy the commutation relations

$$[J_x, J_y] = i\hbar j_z \ etc. \text{ or}[J_i, J_j] = i\hbar\epsilon_{ijk}J_k$$

Since J^2 and any component of **J** commute, though the components of J do not separately commute as noted above, angular momentum eighestates are simultaneously eignestates of J^2 and *e.g.* J_z: ψ_{jm}, where

$$J^2\ \psi_{jm} \ = \ j(j+1)\hbar\ \psi_{jm} \quad J_z\ \psi_{jm} \ = \ m\hbar\ \psi_{jm}$$

It is also useful to make use of raising and lowering operators

$$J_+ = J_x + iJ_y \quad J_- = J_x - iJ_y$$

Which act on $\psi_{j,m}$ as follows:

$$J_\pm\psi_{j,m} = \hbar\sqrt{(j \mp m)(j+1\pm m)}\psi_{j,m\pm1} \quad \text{and} J_+\psi_{j,j} = 0 \quad J_-\psi_{j,-j} = 0$$

The wavefunctions $\psi_{j,m}$ are linear combinations of eigensates of **L** and **s** with coefficients given by the Clebsch-Gordan coefficients, and the eigenstates of s^2 and s_z for $s = 1/2$ can be represented as two component Pauli spinors:

$$\psi_{(s=1/2,m_s=+1/2)} = \begin{pmatrix}1\\0\end{pmatrix} \quad \psi_{(s=1/2,m_s=-1/2)} = \begin{pmatrix}0\\1\end{pmatrix}$$

[1]

[1] Consult your favorite QM textbook, such as Griffin.

For indistinguishable bosons, particles with integer spin, the wave function must be symmetric under exchange of any two particles, *i.e.* $\psi(1,2) = +\psi(2,1)$ for two particles. For fermions, particles with half interger spin, the wave function must be antisymmetric under exchange of any two particles, *i.e.* $\psi(1,2) = -\psi(2,1)$. Here, the meanings of $\psi(1,2)$ and $\psi(2,1)$ in spatial and spin coordinates are

$$\psi(1,2) = \psi(\mathbf{x}^1, s_z^1; \mathbf{x}^2, s_z^2)$$

$$\psi(2,1) = \psi(\mathbf{x}^2, s_z^2; \mathbf{x}^1, s_z^1)$$

For example for two indistinguishable spin 1/2 fermions, the total spin is $\mathbf{s} = \mathbf{s_1} + \mathbf{s_2}$, where $s = 0$ or 1. For $s = 1$, there are three possible values of $m_s = -1, 0, +1$ and the wave function for the spin coordinates must be symmetric under exchange of the two particles since, for example, the $m_s = +1$ state

$$\psi_{(s=1,m_s=1)} = \psi^1_{(s=1/2,m_s=1/2)}\psi^2_{(s=1/2,m_s=1/2)}$$

which does not change under exchange of particles labeled 1 and 2. The symmetric $m_s = 0$ state is

$$\psi_{(s=1,m_s=0)} = \frac{1}{\sqrt{2}}\psi^1_{(s=1/2,m_s=1/2)}\psi^2_{(s=1/2,m_s=-1/2)} + \frac{1}{\sqrt{2}}\psi^1_{(s=1/2,m_s=-1/2)}\psi^2_{(s=1/2,m_s=+1/2)}$$

The orthogonal $m_s = 0$ state corresponds to $s = 0$

$$\psi_{(s=1,m_s=0)} = \frac{1}{\sqrt{2}}\psi^1_{(s=1/2,m_s=1/2)}\psi^2_{(s=1/2,m_s=-1/2)} - \frac{1}{\sqrt{2}}\psi^1_{(s=1/2,m_s=-1/2)}\psi^2_{(s=1/2,m_s=+1/2)}$$

which is antisymmetric under exchange. The spin symmetry of the two particle wavefunction is given by

$$(-1)^{s+1}$$

(Note that the spinor algebra for isospin $I = 1/2$ is identical.)

For a product wavefunction describing several observables such as momentum and spin or spin and isospin, or siospin and charge conjugation, it is the total product wavefunction that must reflect the symmetry of its constituent fermions or bosons. Generally, the constituents are spin 1/2 fermions (quarks or nucleons) and the wavefunction must be antisymmetric. The symmetry of the orbital angular momentum part of a two particle wavefunction follows from reversing the vector from particle 1 to particle 2, that is it is equivalent to the parity operation. Therefore the spatial symmetry of wavefunctions is given by

$$(-1)^L$$

2.2 Symmetry Transformations

A transformation is an operation on the wavefunction that reflects the act of translating, rotating, parity reflection, etc. A very general and powerful sort of transformation is a gauge transformatin familiar in electromagentism. Symmetry is the invariance of the physics, that is the set of observables, under the transformation. In quantum mechanics, we will be interested in Unitary transformations, that is transformations that preserve the quantity $\psi^*\psi$ or $\int \psi^*\psi \, d^3x$.

Consider the electromagnetic gauge transformation of the scalar and vector potentials that preserves the fields **E** and **B**:

$$\rightarrow \phi' = \phi - \frac{d\lambda}{dt} \quad \mathbf{A} \rightarrow \mathbf{A}' = \mathbf{A} + \nabla\lambda$$

The eigenfunctions of the hamiltonian for the potential $\{\phi, \mathbf{A}\}$ and $\{\phi', \mathbf{A}'\}$ differ only by a phase that can be written as

$$\psi' = U\psi = e^{iQ\lambda/\hbar}$$

where U is a unitary operator and q is the eigenvalue of the charge operator Q. We say that the (gauge) transformation (or any symmetry transformation) is *generated* by the operator Q (or **p** or **s** etc.).

The Schrödinger equation for $\psi' = U\psi$ is also transformed. To see what happens, we use the property that $U(\lambda)$ has and inverse $U(\lambda)^{-1}$ such that $U(\lambda)^{-1}U(\lambda) = I$. Starting with $H\psi = E\psi$

$$HU(\lambda)^{-1}U(\lambda)\ \psi = E\ \psi$$

Operating on both sides with $U(\lambda)$, we have

$$U(\lambda)HU(\lambda)^{-1}U(\lambda)\ \psi = E\ U(\lambda)\psi$$

Thus the transformed equation is $H'\ \psi' \ = \ E\ \psi'$ where

$$H' \ = \ U(\lambda)HU(\lambda)^{-1}$$

Note that the set of transformations $U(\lambda)$ form a group in the mathematical sense. A group is a set of transformations or elements that satisfy the following:
1. The product $U(\lambda_1)U(\lambda_2)$ is also an element of the group. $U(\lambda_1)U(\lambda_2)$ is not necessarily equal to $U(\lambda_2)U(\lambda_1)$
2. The associative law holds, *i.e* $(U(\lambda_1)U(\lambda_2))U(\lambda_3) \ = \ U(\lambda_1)(U(\lambda_2)U(\lambda_3))$
3. There is a SINGLE identity element I.
4. Each element has an inverse such that $U(\lambda)^{-1}U(\lambda) = U(\lambda)U(\lambda)^{-1} = I$.

Noether's Theorem states that the observable corresponding to any operator that generates a symmetry transformation leaving H unchanged is a conserved quantity. Thus, for inifinitesimal gauge transformations

$$U\ H\ U^{-1} = (1 + \frac{i}{\hbar}\lambda Q)H((1 - \frac{i}{\hbar}\lambda Q) = H + \frac{i}{\hbar}\lambda(QH - HQ)$$

Thus $\delta\ H = \frac{i}{\hbar}\lambda[Q, H]$ vanishes only if $[Q, H] = 0$ that is if $Q$is constant. Conservation of electric charge therefore requires gauge invariance.